

Experimental Study of Lyapunov Equation Solution Bounds for Power Systems Models

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Abstract: *This research work investigates the applicability of some lower and upper, matrix and scalar bounds for the solution of the Continuous Algebraic Lyapunov Equation (CALE), when the coefficient matrices are the state matrices of real data models. The bounds are illustrated for two different models describing the dynamic behavior of power systems – a two-area power system and an interconnected power system. Some important conclusions referring to the accuracy of the respective estimates are made, as well.*

Keywords: *Lyapunov equation, solution bounds, power systems.*

1. Introduction

The problem of deriving bounds for the solution of the Continuous Algebraic Lyapunov Equation (CALE) attracts interest for more than half a century. This is due to both theoretical and practical reasons. In some cases, due to its high order, the direct solution of this equation is impossible, and in others it is sufficient to have at disposal only some estimates for it. The main difficulty arises from the fact, that the available upper bounds are valid under some assumed restrictions imposed on the coefficient matrix. Due to this, valid solution bounds are possible only for some special subsets of negative stable (Hurwitz) coefficient matrices.

The main purpose of this research is to investigate the quality of some available lower and upper, matrix and scalar bounds for the solution of the CALE. Two state space models with real data of a relatively high order n of power systems

are used as test examples. The first one is a model of a two-area power system with $n = 11$ states and the second one describes the dynamics of an interconnected power system with four local subsystems and $n = 8$ states. Lower and upper matrix, eigenvalue and trace estimates are obtained for the solution. They are compared with the exact values and some important conclusions referring to their accuracy are drawn.

The following notations will be used: $A > 0$ ($A \geq 0$) indicates that A is a positive (semi-positive) definite matrix; $A \in \mathbf{R}_{n,m}$ ($\mathbf{R}_{n,n} \equiv \mathbf{R}_n$) and $a \in \mathbf{R}^n$ denote a real $n \times m$ matrix A and $n \times 1$ vector a ; $\text{tr}(A)$, $A^{1/2}$, A^{-1} , A^T are the trace (sum of eigenvalues), the square root (if A is positive semi-definite), the inverse (if A is nonsingular) and the transpose of matrix A ; the symmetric part of matrix A is denoted $A_s = A^T + A$; $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimal and maximal eigenvalue of a symmetric matrix A respectively; the identity matrix of respective dimension is I .

2. Bounds for the solution of the CALE

From Lyapunov's stability theorem it follows that a matrix A is Hurwitz (negative stable) if and only if the CALE

$$(1) \quad A^T P + PA = -Q, \quad A \in \mathbf{R}_n$$

has a unique positive definite solution matrix P for any given positive definite matrix Q . Due to various reasons, having at disposal only bounds on P can be sufficient in some practical control design and analysis problems.

It must be emphasized on the fact that the upper bounds are always more difficult to derive, since they are valid only under some restrictions. Till 2004 all available upper bounds for the solution of the CALE (e.g., [1] and [2]) are valid under the very conservative condition that the symmetric part of the coefficient matrix A is negative definite. By making use of the singular value decomposition of A , i.e.,

$$A = U \Sigma V^T = (U \Sigma U^T) U V^T = (A A^T)^{1/2} F, \\ U U^T = V V^T = F F^T = I, \quad \Sigma = \text{diag}\{\sigma_i\} > 0, \quad i = 1, \dots, n,$$

it was proved in [5] that

$$(2) \quad A_s < 0 \Rightarrow F_s = A^T R + R A < 0, \quad R = (A A^T)^{-1/2}.$$

This simple fact helped to extend the set of stable coefficient matrices for which there exist valid upper bounds. Various types of based on this result bounds were derived in [3] and [4].

Definition 1. A matrix $L > 0$ is said to be a Lyapunov Matrix (LM) for A if $A^T L + L A < 0$.

In this sense, if the symmetric part of A is negative definite, then I and R are LMs for A , in accordance with (2), but R can be a LM even if this condition does not hold.

If there exist positive definite matrices P_l, P_u , such that

$$(3) \quad A^T P_u + P_u A \leq -Q \leq A^T P_l + P_l A \Rightarrow 0 < P_l \leq P \leq P_u,$$

i.e., P_l, P_u are lower and upper matrix bounds for P , respectively. In other words, the solution estimation problem can always be solved if such matrices can be obtained. On the other hand, their determination in the general case, e.g., by LMI solution, may require computational effort comparable with the one needed for the direct solution of the Lyapunov Equation (LE) and therefore should be avoided.

Theorem 1. Let R in (2) be a LM for A . Then the solution P of the LE (1) has the following bounds:

$$(4) \quad \mu_l R = P_l \leq P \leq P_u = \mu_u R, \quad \mu_l = \lambda_m(-QF_s^{-1}), \quad \mu_u = \lambda_M(-QF_s^{-1}),$$

$$(5) \quad \lambda_m(P_l) \leq \lambda_m(P) \leq \lambda_l(P) \leq \lambda_M(P) \leq \lambda_M(P_u), \quad i = 1, \dots, n,$$

$$(6) \quad \text{tr}(P_l) \leq \text{tr}(P) \leq \text{tr}(P_u).$$

Proof: Let the above assumption holds, i.e., the symmetric part of matrix F is negative definite. From the definition in (4) of the scalar and matrix parameters μ_l, μ_u, P_l, P_u , it follows that

$$\begin{aligned} \mu_l &= \lambda_m[(-F_s)^{1/2} Q (-F_s)^{1/2}] \Rightarrow \mu_l I \leq \\ &\leq (-F_s)^{1/2} Q (-F_s)^{1/2} \Rightarrow -\mu_l F_s \leq Q \Rightarrow -Q \leq \mu_l F_s, \\ \mu_u &= \lambda_M[(-F_s)^{1/2} Q (-F_s)^{1/2}] \Rightarrow \mu_u I \geq \\ &\geq (-F_s)^{1/2} Q (-F_s)^{1/2} \Rightarrow -\mu_u F_s \geq Q \Rightarrow -Q \geq \mu_u F_s. \end{aligned}$$

Having in mind that $F_s = A^T R + RA$, one gets the matrix inequalities in (3) for P_l, P_u defined as in (4). This proves the matrix bounds (4). The eigenvalue and trace bounds (5) and (6) follow from the well known facts that if $X \leq Y \leq Z$, then

$$\lambda_i(X) \leq \lambda_i(Y) \leq \lambda_i(Z), \quad i = 1, \dots, n, \quad \text{tr}(X) \leq \text{tr}(Y) \leq \text{tr}(Z)$$

for arbitrary $n \times n$ symmetric matrices X, Y and Z . ■

3. Power systems models

Large-scale power systems consist of a number of interconnected via tie-lines power control areas. Different complicated nonlinear models of such a system are available. However, for control design purposes a simplified linearized model is usually used. A two-area power system is taken as a test system in this study. The generators are assumed to be a coherent group in each area, which includes a governor, a reheater and a steam turbine.

3.1. Two-area power system

The set of first order differential equations with constant coefficients governing the overall process in a two-area power system and the description of the respective variables and coefficient parameters is given below [6]:

$$\begin{aligned} \dot{P}_{C1} &= k_{I1} b_1 F_1 + k_{I1} P_{tie}, \\ \dot{X}_{G1} &= -(1/t_{G1}) P_{C1} - (1/t_{G1}) X_{G1} - (1/r_1 t_{G1}) F_1 + (1/t_{G1}) u_{21}, \end{aligned}$$

$$\begin{aligned}
\dot{P}_{R1} &= (1/t_{T1})X_{G1} - (1/t_{T1})P_{R1}, \\
\dot{P}_{T1} &= (k_1/t_{T1})X_{G1} + aP_{R1} - (1/t_{R1})P_{T1}, \\
\dot{F}_1 &= (k_{p1}/t_{p1})P_{T1} - (1/t_{p1})F_1 - (k_{p1}/t_{p1})P_{\text{tie}} - (k_{p1}/t_{p1})u_{11}, \\
\dot{P}_{\text{tie}} &= 2\pi T_{12}F_1 - 2\pi T_{12}F_2, \\
\dot{P}_{C2} &= -k_{I2}F_1 - k_{I2}P_{T2}, \\
\dot{X}_{G2} &= -(1/t_{G2})P_{C2} - (1/t_{G2})X_{G2} - (1/r_2 t_{G2})F_2 + (1/t_{G2})u_{22}, \\
\dot{P}_{R2} &= (1/t_{T2})X_{G2} - (1/t_{T2})P_{R2}, \\
\dot{P}_{T2} &= (k_2/t_{T2})X_{G2} + bP_{R2} - (1/t_{R2})P_{T2}, \\
\dot{F}_2 &= (k_{p2}/t_{p2})P_{\text{tie}} - (1/t_{p2})F_2 + (k_{p2}/t_{p2})P_{T2} - (k_{p2}/t_{p2})u_{12},
\end{aligned}$$

where

P_{Ci} – incremental change in the integral controller i ,

X_{Gi} – fictitious state variable,

P_{Ri} – incremental change in the output energy of the i -th reheat type turbine in MW,

P_{Ti} – incremental change in the output of the i -th subsystem in MW,

F_i – incremental frequency deviation in subsystem i in Hz,

P_{tie} – incremental change in the tie-line power,

t_{Gi} – i -th governor time constant in s,

t_{Ti} – i -th turbine time constant in s,

t_{Ri} – i -th reheat time constant in s,

t_{Pi} – i -th subsystem time constant in s,

k_{Pi} – i -th subsystem gain in Hz/MW

k_{Ii} – i -th subsystem integral control gain,

b_i – i -th subsystem frequency-biasing factor in MW/Hz

k_i – ratio between the output energy of the i -th turbine to the total output energy,

r_i – speed regulation for the i -th subsystem due to the governor action in Hz/MW,

t_{ij} – synchronizing coefficient of the tie-line power between subsystems i and j .

The nominal values of the parameters are given in Appendix 1. Having in mind the above set of differential equations and defining the state and control vectors $x(t)$ and $u(t)$, respectively, as follows

$$\begin{aligned}
x(t)^T &= (P_{C1} \ X_{G1} \ P_{R1} \ P_{T1} \ F_1 \ P_{\text{tie}} \ P_{C2} \ X_{G2} \ P_{R2} \ P_{T2} \ F_2), \\
u(t)^T &= (u_{11} \ u_{12} \ u_{21} \ u_{22})
\end{aligned}$$

helps to put the model in the well known compact matrix (state space) form

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(t) \in \mathbf{R}^{11}, \quad x(0) = x_0, \quad u(t) \in \mathbf{R}^4.$$

It has been concluded that the nominal open loop coefficient matrix A is not stable, since it has eigenvalues with positive real parts. An optimal stabilizing state control law $u(t) = -Kx(t)$ has been applied to the system, where $K = -B^T R$ and R is the positive definite solution of the continuous algebraic Riccati equation $A^T R + RA - PBB^T R = -I$. The stable close loop nominal system is $\dot{x} = A_c x$, $A_c = A - BK$. The nominal systems parameters and the computed for them state matrix are given in Appendix 1.

3.2. Interconnected power system

Consider the linearized model of an interconnected power system comprised of N local generators described by the set of differential equations [7]:

$$\Delta \dot{\omega}_i = -R_i^{-1} \Delta \omega_i + \sum_{j=1}^N Y_{ij} \delta_j, \quad Y_{ij} = Y_{ji}, \quad i = 1, 2, \dots, N,$$

$$\dot{\delta}_i = \Delta \omega_i, \quad i = 1, 2, \dots, N,$$

where ω_i and δ_i , $i = 1, 2, \dots, N$, denote the angular speed and deviation of the i -th rotor, Y_{ij} is the transfer conductance between subsystems i and j , Y_{ii} is the self conductance of the i -th generator and R_i is a specific coefficient associated with the i -th subsystem. Using the notation $x = (x_1^T \ x_2^T \ \dots \ x_i^T \ \dots \ x_N^T)^T \in \mathbf{R}^{2N}$, $x_i = (\Delta \omega_i \ \delta_i)^T \in \mathbf{R}^2$, the above model can be rewritten in a compact matrix form, where the state matrix has the following typical structure

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{12} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{1N} & A_{2N} & \dots & A_{NN} \end{bmatrix} \in \mathbf{R}_{2N};$$

$$A_{ii} = \begin{bmatrix} -R_i^{-1} & Y_{ii} \\ 1 & 0 \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 0 & Y_{ij} \\ 0 & 0 \end{bmatrix} = A_{ji} \in \mathbf{R}_2.$$

The considered system is comprised of $N = 4$ subsystems with the following nominal parameter values

$$R_i = 0.01, \quad i = 1, \dots, 4, \quad Y_{11} = -2.2, \quad Y_{22} = -2.4, \quad Y_{33} = -2.6,$$

$$Y_{44} = -2.8, \quad Y_{12} = 0.5, \quad Y_{13} = 0.3, \quad Y_{14} = 0.2, \quad Y_{23} = 0.6, \quad Y_{24} = 1, \quad Y_{34} = 0.1.$$

The nominal stable state matrix A is given in Appendix 2.

4. Numerical experiments

4.1. Two-area power system

It is required to determine lower and upper, matrix and scalar bounds for the solution P of the LE, where the coefficient matrix in (1) is A_c . The solution P provides also some valuable information about the quality of the transient process, since with the close loop system an integral performance index can be associated of the form

$$(7) \quad J = \int_0^{\infty} x^T(t) Q x(t) dt = x_0^T P x_0$$

which can be bounded from below and above by respective estimates (if available), as well, i.e.,

$$x_0^T P_l x_0 = J_l \leq J \leq J_u = x_0^T P_u x_0$$

which helps to estimate *a priori* the dynamic behavior of the system for any given non-zero initial state vector without solving the CALE.

Since the symmetric part of the coefficient matrix A_c is not negative definite (it contains zero diagonal entries), all upper bounds based on this assumption are not valid in this case. But R is found to be a LM for it and the bounds in (4), (5) and (6) are all valid.

The LE (1) is solved for $Q = I$. The scalar parameters in (4) are computed as $\mu_l = 0.5$, $\mu_u = 1.991$. The comparison process includes the analysis of the extremal eigenvalues and the traces of the exact solution matrix P and their lower and upper estimates. All eleven eigenvalues (given in a descending order) and traces of the solution P and its lower and upper matrix bounds (4), are given in Table 1 and Table 2.

Table 1

Eigenvalues	P_l	P_{exact}	P_u
1	115.4130	178.4124	441.3163
2	15.2589	24.6369	58.3469
3	0.9310	1.8672	3.5598
4	0.3133	0.5402	1.1981
5	0.1189	0.1403	0.4546
6	0.1119	0.1323	0.4280
7	0.0502	0.0535	0.1920
8	0.0256	0.0341	0.0979
9	0.0221	0.0326	0.0844
10	0.0034	0.0082	0.0131
11	0.0061	0.0081	0.0232

Table 2

$\text{tr}(P_l)$	$\text{tr}(P)$	$\text{tr}(P_u)$	$\bar{\lambda}_l$	$\bar{\lambda}$	$\bar{\lambda}_u$
132.2544	205.8658	505.7143	12.0231	18.7151	45.974

If $\text{tr}(X)$ is the trace of some $n \times n$ positive definite matrix X , then the parameter $\bar{\lambda} = \text{tr}(X)/n$ is introduced to define the ‘‘average’’ eigenvalue of X . It becomes clear that the lower eigenvalue and trace bounds for P are more precise than the upper ones, which is not surprising. The accuracy of the bounds depends entirely on the parameters μ_l, μ_u , since $\Delta P = P_u - P_l = \mu R$, $\mu = \mu_u - \mu_l$.

The smaller is the difference, the tighter are all bounds, since the matrix interval containing the solution P becomes narrower.

Now, it will be interesting to get some additional estimates. Define the ‘‘average’’ matrix estimate as $E = 0.5(P_l + P_u)$. It is clear that $P_l \leq E \leq P_u$, i.e., E belongs to the matrix interval which contains the exact solution P . The estimate error matrix is $\Delta = P_u - P_l$. The following results for the eigenvalues, traces and average eigenvalues of these matrices are computed:

$$\lambda_m(E) = 0.0146, \lambda_M(E) = 278.365, \text{tr}(E) = 318.984, \bar{\lambda}(E) = 28.9986;$$

$$\lambda_m(\Delta) = 0.0171, \lambda_M(\Delta) = 325.9, \text{tr}(\Delta) = 373.46, \bar{\lambda}(\Delta) = 33.9509.$$

Consider the performance index (7). It is clear that the following scalar bounds are valid for any initial state $x_0^T P_l x_0 = J_l \leq J \leq J_u = x_0^T P_u x_0$. Let x_0 be an 11×1 vector with all entries equal to one. Then, J represents the sum of all entries of the solution matrix P . The following results are obtained:

$$144.7214 = J_l \leq J = 221.6909 \leq J_u = 553.3855;$$

$$x_0^T E x_0 = 349.05, x_0^T \Delta x_0 = 408.6642.$$

It can be verified that the matrix bounds and their parameters provide rather good estimates for the exact solution, which can be characterized by a very high value for the ratio $\lambda_M(P)/\lambda_m(P) = 22026$.

Naturally it is expected that the bounding interval is a rather extended one in such cases.

4.2. Interconnected power system

The LE (1) is solved for A , as given in Appendix 2 and $Q = I$. The scalar parameters in (4) are computed as $\mu_l = 0.50001537$, $\mu_u = 0.5007855$, which supposes rather tight solution estimates. The extremal eigenvalues, traces and average eigenvalues of P and its lower and upper matrix bounds (4) are given below:

$$\lambda_m(P) = 0.0050007, \lambda_M(P) = 33.81064, \text{tr}(P) = 77.078, \bar{\lambda}(P) = 9.6347,$$

$$\lambda_m(P_l) = 0.004998, \lambda_M(P_l) = 33.81063, \text{tr}(P_l) = 77.046, \bar{\lambda}(P_l) = 9.6308,$$

$$\lambda_m(P_u) = 0.005436, \lambda_M(P_u) = 33.853348, \text{tr}(P_u) = 77.643, \bar{\lambda}(P_u) = 9.7504.$$

The percentage errors in the respective estimates are summarized as it is shown in Table 3.

Table 3

$\Delta\lambda_m(P_l)$	$\Delta\lambda_M(P_l)$	$\Delta\lambda_m(P_u)$	$\Delta\lambda_M(P_u)$	Δtr_m	Δtr_M	$\Delta\bar{\lambda}(P_l)$	$\Delta\bar{\lambda}(P_u)$
%	%	%	%	%	%	%	%
0.054	3×10^{-5}	0.0732	0.1263	0.04	0.16	0.0405	0.7335

All respective errors are less than 0.13 %, which confirms that all estimates are very tight in this case. This fact is also clear from the computed values for the extremal eigenvalues, traces and average eigenvalues of the average matrix estimate $E = 0.5(P_l + P_u)$ and the error matrix $\Delta = P_u - P_l$:

$$\lambda_m(E) = 0.005001, \lambda_M(E) = 33.832, \text{tr}(E) = 77.095, \bar{\lambda}(E) = 9.6369,$$

$$\lambda_m(\Delta) = 0.000006, \lambda_M(\Delta) = 0.043, \text{tr}(\Delta) = 0.0973, \bar{\lambda}(\Delta) = 0.121.$$

The performance index and its lower and upper estimates are computed for the same initial state vector as in example 4.1, i.e., $128.99 = J_l \leq J = 129 \leq J_u = 129.16$, which also illustrates the fact that the obtained solution bounds are rather precise in this case.

References

1. Fang, Y., K. Loparo, X. Feng. New Estimates for Solutions of Lyapunov Equations. – IEEE Transactions on Automatic Control, Vol. **42**, 1997, No 3, 408-411.
2. Komaroff, N. Upper Sumation and Product Bounds for Solution Eigenvalues of the Lyapunov Matrix Equation. – IEEE Transactions on Automatic Control, Vol. **37**, 1992, No 5, 337-341.
3. Savov, S., I. Popchev. Bounds for the Solution of the Continuous Lyapunov Equation under Relaxed Validity Constraints. – Advanced Studies in Contemporary Mathematics, Vol. **18**, 2009, No 2, 235-243.
4. Savov, S., I. Popchev. New Generalized Upper Trace Bound for the Solution of the Lyapunov Equation. – International Journal of Pure and Applied Mathematics, Vol. **49**, 2008, No 3, 381-389.
5. Savov, S., I. Popchev. New Upper Estimates for the Solution of the Continuous Algebraic Lyapunov Equation. – IEEE Transactions on Automatic Control, Vol. **49**, 2004, No 10, 1841-1842.
6. Shayeghi, H., H. Shayanfar. Automatic Generation Control of Interconnected Power Systems Using ANN Technique Based on μ -Analysis. – Journal of Electrical Engineering, Vol. **55**, 2004, No 11-12, 306-313.
7. Siljak, D. Decentralized Control of Complex Systems. Boston, Academic Press, 1991.

Appendix 1

The nominal values of the system's parameters are as follows:

$$t_{G1} = t_{G2} = 0.1 \text{ s}; t_{T1} = t_{T2} = 0.3 \text{ s}; t_{R1} = t_{R2} = 10 \text{ s};$$

$$t_{P1} = t_{P2} = 20 \text{ s}; k_{P1} = k_{P2} = 120 \text{ Hz/MW};$$

$$\begin{aligned}
r_1 = r_2 = 2.4 \text{ Hz/MW}; b_1 = b_2 = 0.425 \text{ MW/Hz}; \\
k_1 = k_2 = 0.5; k_{T1} = k_{T2} = 0.05; T_{12} = 0.0707, a_{12} = 1; \\
a = (k_1 + k_2)/t_{T1} - k_1/t_{R1}, b = (k_1 + k_2)/t_{T2} - k_2/t_{R2}.
\end{aligned}$$

The stabilized close loop system state matrix $A_c = A - BK$ is partitioned as follows:

$$A_c = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad A_{11} \in \mathbf{R}_5, \quad A_{22} \in \mathbf{R}_6, \quad A_{12} \in \mathbf{R}_{5,6}, \quad A_{21} \in \mathbf{R}_{6,5};$$

$$A_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.0213 \\ -6.0178 & -16.5623 & -5.7229 & -12.2171 & -3.7546 \\ 0 & 3.3333 & -3.3333 & 0 & 0 \\ 0 & 1.6666 & 1.6667 & -0.1 & 0 \\ -74.0812 & 12.8135 & 12.032 & 33.8268 & -60.9484 \end{bmatrix};$$

$$A_{22} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0213 & 0 \\ 0.4513 & -12.4877 & -14.7628 & -2.6993 & -0.5823 & -3.8753 \\ 0 & 0 & 3.3333 & -3.3333 & 0 & 0 \\ 0 & 0 & 1.6666 & 1.6667 & -10 & 0 \\ 76.2689 & 38.86 & 8.4688 & 4.8353 & 19.2739 & -61.4985 \end{bmatrix};$$

$$A_{12} = \begin{bmatrix} 0.05 & 0 & 0 & 0 & 0 & 0 \\ 1.4189 & 0.283 & -0.0186 & -0.028 & -0.0535 & -0.0109 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -72.0432 & 8.5496 & 0.1233 & -0.0173 & -1.1328 & 0.1128 \end{bmatrix};$$

$$A_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.444 \\ 0 & 0 & 0 & 0 & -0.005 \\ 0.1984 & -0.0186 & -0.029 & -0.1529 & 0.0034 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 20.1713 & -0.394 & -0.6452 & -5.9822 & 0.1128 \end{bmatrix}.$$

Appendix 2

The nominal stable state matrix A of the interconnected power system is

$$\begin{bmatrix} -100 & -2.2 & 0 & 0.5 & 0 & 0.3 & 0 & 0.2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & -100 & -2.6 & 0 & 0.6 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0.6 & -100 & -3 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 1 & 0 & 0.1 & -100 & -3.4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$